



## ENTROPY EXCHANGE AND ENTANGLEMENT IN THE JAYNES-CUMMINGS MODEL WITH TRANSIENT EFFECTS

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**ABSTRACT.** In this paper, we investigate the dynamics of entropy exchange and entanglement in the atom-field interaction by the Jaynes-Cummings model in the presence of the transient effects considered for the case of linear sweep. We find that the transient effects do not influence the entropy exchange between the atom and the field. As the strength of these effects increases, the oscillations of the entropy change and entanglement speed up. The entanglement behaves chaotically as the transient effects become stronger

### INTRODUCTION

Cavity Quantum Electrodynamics (CQE) keeps an important place in quantum optics and attracts much attention [1, 2, 3]. Perhaps, the simplest model of CQE is the Jaynes-Cummings Model (JCM) [4]. The model describes the system of a two-level atom interacting with a quantized mode of an optical cavity, with or without the presence of light. In spite of its simplicity, the JCM reveals important properties of light such as the discreteness of field states [1, 5]. The model is open to some extensions to consider additional effects. Some of the extensions are initial conditions [6], dissipation and damping [7, 8, 9], multilevel atoms and multiple atoms [10] and multi-mode description of the field [11]. Another extension of the JCM is incorporation of the transient effects considered for the linear sweep as studied by Joshi and Lawande [12]. Experimentally, this extension can describe an atom entering a cavity subjecting to a very slow shift or a sudden jump of the electric field. Linear sweep model was considered elsewhere [13, 14]. We previously studied the influence of the transient effects on the dynamics of entanglement between a JCM atom and an isolated atom [15]. We showed that the entanglement sudden death can be controlled by these transient effects. Transient effects has also been studied elsewhere [16, 17].

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Boukobza and Tannor [18] studied the JCM for the mixed states of field and atom and showed that there is an entropy exchange between the field and the atom. The entropy exchange dynamics in the atom-field interactions was studied extensively elsewhere [19, 20, 21, 22, 23, 24].

In this paper, we use the model in Ref. [12] to study the dynamical properties of entropy correlations and entanglement in the atom-field interaction in the presence of the transient effects. We show that the transient effects do not influence the dynamical behavior of entropy exchange between the atom and the field. As the strength of these effects increases, the oscillations of the entropy change and entanglement speed up. The entanglement behaves chaotically as the transient effects become stronger.

### SYSTEM AND SOLUTION

The Hamiltonian of the system with the resonance between the atomic transition and the field frequencies is ( $\hbar = 1$ ) [4]

$$H = \omega S_z + \omega a^\dagger a + g(S_+ a + S_- a^\dagger) \quad (1)$$

where  $S_\pm$  and  $S_z$  are spin-1/2 atomic operators,  $a$  and  $a^\dagger$  are the field annihilation and creation operators and also  $g$  is the coupling coefficient between the atom and the field. In order to incorporate the transient effects, the coupling coefficient  $g$  is modified as [12]

$$g(t) = gf(t) \quad (2)$$

where the function  $f(t)$  describes the linear sweep.  $f(t)$  contains the two limiting cases which correspond to sudden jump and adiabatic variation. The cavity-mode quantized field is switched on by a linear ramp described by this function. It is defined as

$$f(t) = \begin{cases} kt/T & \text{for } 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

As the value of  $k$  increases over a fixed time interval  $T$  from small values to large values, the strength of the interaction changes from adiabatic variation to sudden jump.

The atom is initially taken in a mixed state

$$\rho^a(0) = P_e |e\rangle\langle e| + P_g |g\rangle\langle g| \quad (3)$$

with  $P_e + P_g = 1$  and  $0 \leq P_e, P_g \leq 1$  and the field is initially taken in a thermal state

$$\rho^f(0) = \sum_{n=0} P_n |n\rangle\langle n| \quad (4)$$

whose probability distribution  $P_n$  is given by

$$P_n = \frac{1}{(1 + \langle n \rangle)} \left( \frac{\langle n \rangle}{1 + \langle n \rangle} \right)^n \quad (5)$$

where  $\langle n \rangle$  is the initial mean photon number in the cavity. So, the initial state of the total system which is a product state, becomes

$$\rho^{fa}(0) = \rho^f(0) \otimes \rho^a(0) = P_e \sum_{n=0} P_n |ne\rangle \langle ne| + P_g \sum_{n=0} P_n |ng\rangle \langle ng| \quad (6)$$

where  $|ne\rangle = |n\rangle \otimes |e\rangle$  and  $|ng\rangle = |n\rangle \otimes |g\rangle$ . The Hamiltonian operator of our system is time-dependent due to the coupling  $g(t)$ , but the Hamiltonians at different times commute. Then, time-evolution of the total system is obtained by

$$\rho^{fa}(t) = \exp[-i \int_0^t H(\hat{t}) d\hat{t}] \rho^{fa}(0) \exp[i \int_0^t H(\hat{t}) d\hat{t}] \quad (7)$$

Then, we obtain

$$\begin{aligned} \rho^{fa}(t) = & P_e \sum_n P_n [\cos^2(c_{n+1}(t)) |ne\rangle \langle ne| + i \sin(c_{n+1}(t)) \cos(c_{n+1}(t)) |ne\rangle \langle n+1g| \\ & - i \sin(c_{n+1}(t)) \cos(c_{n+1}(t)) |n+1g\rangle \langle ne| + \sin^2(c_{n+1}(t)) |n+1g\rangle \langle n+1g|] \\ & + P_g \sum_n P_n [\cos^2(c_n(t)) |ng\rangle \langle ng| + i \sin(c_n(t)) \cos(c_n(t)) |ng\rangle \langle n-1e| \\ & - i \sin(c_n(t)) \cos(c_n(t)) |n-1e\rangle \langle ng| + \sin^2(c_n(t)) |n-1e\rangle \langle n-1e|] \end{aligned} \quad (8)$$

where  $c_n(t) = g\sqrt{n} \frac{kt^2}{2T}$ . The density matrix of the atom (and the field) can be found by tracing out  $\rho^{fa}(t)$  over the degree of freedom of the field (and the atom). The atomic density matrix becomes

$$\begin{aligned} \rho^a(t) = & \{P_e \sum_n P_n \cos^2(c_{n+1}(t)) + P_g \sum_n P_{n+1} \sin^2(c_{n+1}(t))\} |e\rangle \langle e| \\ & + \{P_g \sum_n P_n \cos^2(c_n(t)) + P_e \sum_n P_{n-1} \sin^2(c_n(t))\} |g\rangle \langle g| \end{aligned} \quad (9)$$

The elements of the density matrix of the field becomes

$$\begin{aligned} \rho_{nn}^f(t) = & P_e \{P_n \cos^2(c_{n+1}(t)) + P_{n-1} \sin^2(c_n(t))\} + \\ & P_g \{P_n \cos^2(c_n(t)) + P_{n+1} \sin^2(c_{n+1}(t))\} \end{aligned} \quad (10)$$

## RESULTS

We now numerically investigate the entropy correlations and the entanglement between the atom and the field by the figures. (In these, we assume that  $g = 1$ .) For our computations, we truncate the series when  $\sum_{n=0} P_n \approx 1$  [18]. For the entropy correlations, we compute the von-Neumann entropy of the atom and the

field subsystems. The entropy of a system is defined as

$$S = - \sum_i \lambda_i \log \lambda_i \quad (11)$$

where  $\lambda_i$ s are the non-zero eigenvalues of the relevant density matrix. The entropy change ( $\Delta S = S(t) - S(0)$ ) for the atom and for the field can be computed by using their density matrices described in Eqs. (9) and (10).

Figs. (1)-(5) show the time-evolution of entropy changes of the atom and the field and their entanglement in the presence of the transient effects. Numerically,  $k = 0.5$  may describe an adiabatic variation and  $k = 8.0$  may describe a sudden jump in the interaction. As we move from the adiabatic variation case to the sudden jump case (the value of  $k$  increases), the periodic evolution of entropy changes and entanglement speeds up. In Fig. (1), the atom is in the excited state and the field is in a weakly excited thermal state with the average photon number  $\langle n \rangle = 0.1$ . The entropy changes of the atom and the field fluctuate together and so they are correlated. When the atom is taken to be in the ground state as shown in Fig. (2), the sum of the atom and field entropy changes is quasi-conserved. Their entropy relations are anti-correlated. There is an entropy exchange between the atom and the field, although the exchange is not complete. When the atomic state is close to ground state ( $P_g = 0.9$ ), there is an almost complete entropy exchange, as shown in Fig. (3). The sum of the atom and field entropy changes is almost completely conserved. The transient effects do not have any influence on the behavior of the entropy exchange between the atomic and the field states. The sum of the entropy changes of the atom and the field is approximately zero in the presence of the transient effects.

For the entanglement properties of the system, we calculate a lower bound on concurrence (LBC) such that the joint state of the atom-field system  $2 \otimes \infty$  is projected onto  $2 \otimes 2$  systems by means of the projection operator

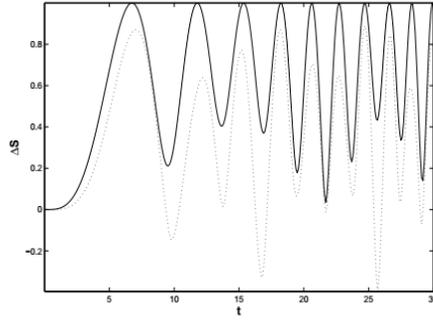
$$\Pi_n = (|g\rangle\langle g| + |e\rangle\langle e|) \otimes (|n\rangle\langle n| + |n+1\rangle\langle n+1|) \quad (12)$$

The resulting density operator is

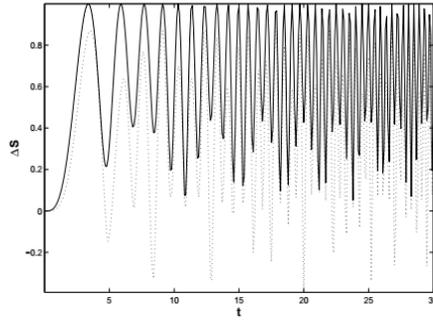
$$\rho_n(t) = \frac{1}{T_n(t)} \Pi_n \rho^{fa}(t) \Pi_n \quad (13)$$

where  $T_n(t) = \text{Tr}(\Pi_n \rho^{fa}(t) \Pi_n)$  denotes the probability of obtaining  $\rho_n(t)$  which is a sub-state with the dimension  $2 \otimes 2$ . For the  $2 \otimes 2$  systems, the degree of entanglement can be quantified by the Wootters' concurrence [25]. Concurrence varies from 0 for the separable states to 1 for the maximally entangled states. Entanglement of the total system can be quantified by averaging over the entanglement of all the sub-states of the system [26]

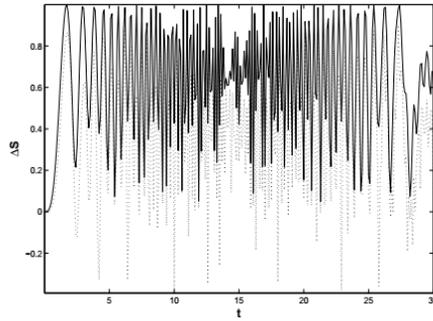
$$C(t) = \frac{\sum_n T_n(t) C_n(t)}{\sum_n T_n(t)} \quad (14)$$



(a)

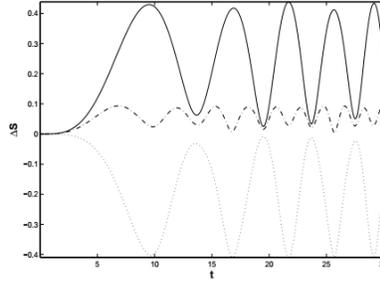


(b)

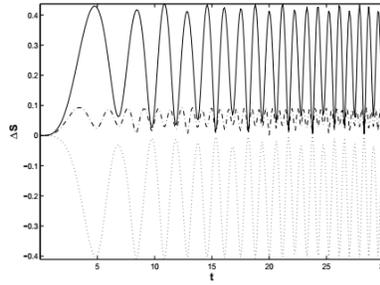


(c)

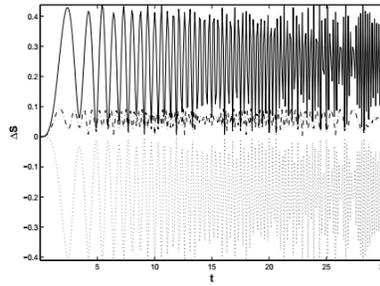
FIGURE 1. Entropy change  $\Delta S$  for the atom (solid line) and for the field (dot line) as a function of time  $t$ .  $P_e = 1$ ,  $\langle n \rangle = 0.1$  and  $T = 30$ . (a)  $k = 0.5$  (b)  $k = 2.0$  (c)  $k = 8.0$ .



(a)



(b)



(c)

FIGURE 2. Entropy change  $\Delta S$  for the atom (solid line), for the field (dot line) and for their sum (dash-dot line) as a function of time  $t$ .  $P_g = 1$ ,  $\langle n \rangle = 0.1$  and  $T = 30$ . **(a)**  $k = 0.5$  **(b)**  $k = 2.0$  **(c)**  $k = 8.0$ .

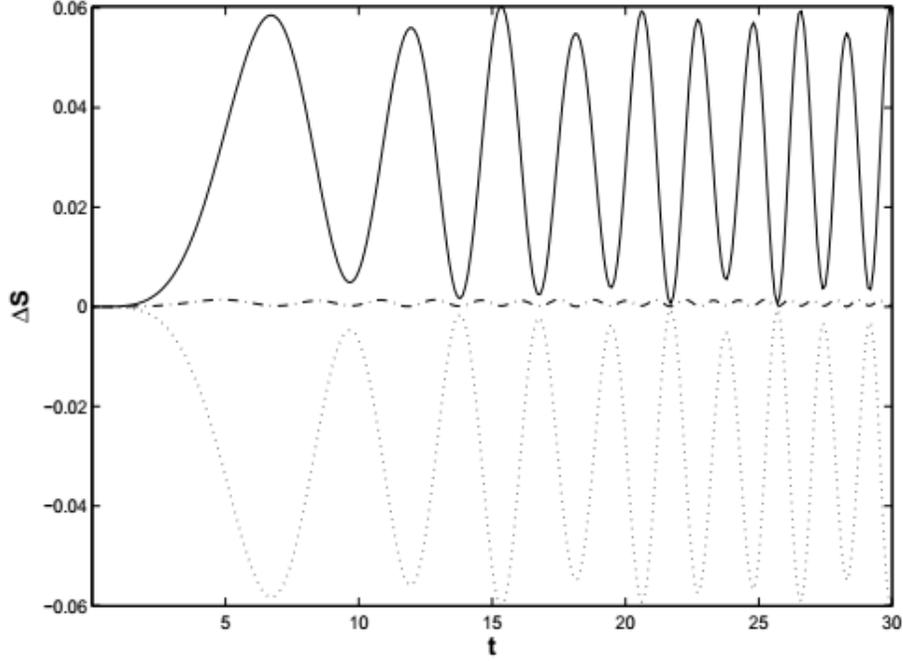


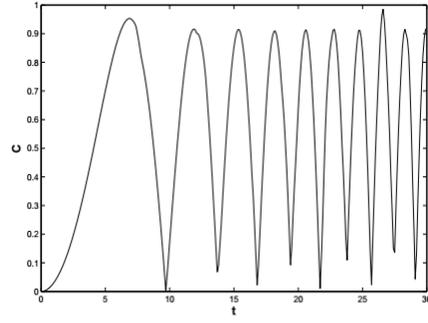
FIGURE 3. Entropy change  $\Delta S$  for the atom (solid line), for the field (dot line) and for their sum (dash-dot line) as a function of time  $t$ .  $P_g = 0.9$ ,  $\langle n \rangle = 0.1$ ,  $T = 30$  and  $k = 1.0$ .

where  $C_n(t)$  is the concurrence of sub-state  $\rho_n(t)$ .

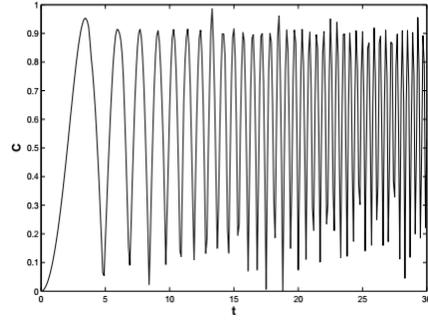
Fig. (4) shows the time-evolution of the entanglement of the system for  $P_e = 1$ . It is obvious that the time-elapsd for the collapse and recovery of the entanglement is long around the beginning of the interaction and then shortens, as time passes. Also, there is a time interval (around  $t \approx 15$ ) at which the amplitude of the oscillations are relatively suppressed. So, there is some chaos at these instants of time in the interaction. We now calculate the time-average of the concurrence to see more clearly this chaotic influence of the transient effects on the entanglement. We compute it as

$$C_{av} = \frac{\int_0^T C(t) dt}{T} \quad (15)$$

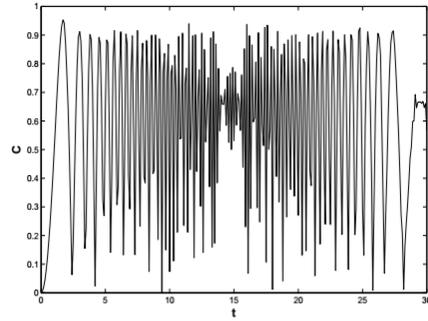
where we take  $T = 30$  and take the integral numerically.



(a)



(b)



(c)

FIGURE 4. Concurrence  $C$  as a function of time  $t$ .  $P_e = 1$ ,  $\langle n \rangle = 0.1$  and  $T = 30$ . (a)  $k = 0.5$  (b)  $k = 2.0$  (c)  $k = 8.0$ .

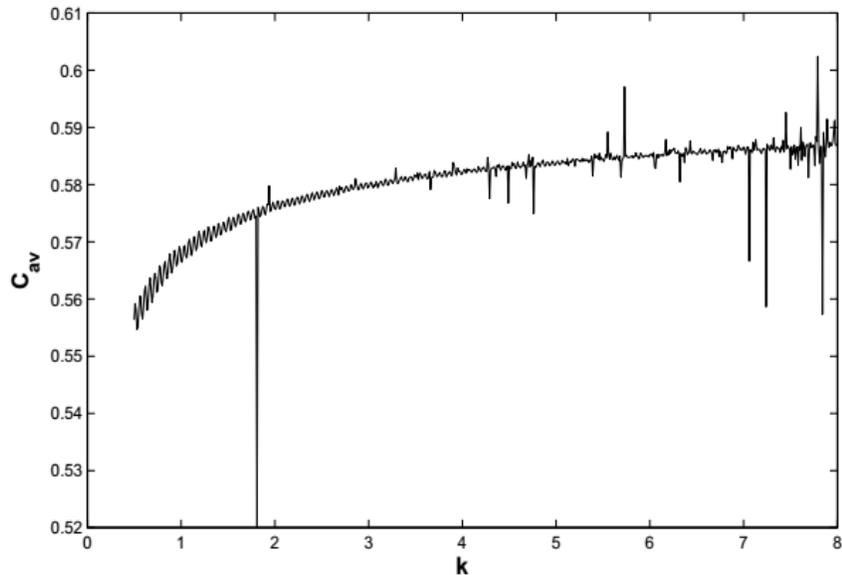


FIGURE 5. Average Concurrence  $C_{av}$  as a function of the parameter of the transient effects  $k$ .  $P_e = 1$ ,  $\langle n \rangle = 0.1$  and  $T = 30$ .

Fig. (5) shows the evolution of the time-averaged entanglement as a function the parameter of the transient effects. It is clear that the average entanglement fluctuates chaotically with the parameter  $k$ . There are unexpectedly considerable sharp rises and falls at some values of  $k$ . What we normally expect is the certain monotonic increase of the average entanglement with the parameter  $k$ . Apart from these random fluctuations, the higher values of  $k$ , the higher magnitude of the entanglement, as expected.

#### CONCLUSION

In summary, we have examined the dynamics of the entropy changes and the entanglement in the atom-field interaction by the Jaynes-Cummings model in the presence of the transient effects. We have showed that the transient effects do not influence the dynamical behavior of entropy exchange between the atom and the field. As the strength of these effects increases, the oscillations of the entropy change and entanglement speed up. The entanglement behaves chaotically as the transient effects become stronger.

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